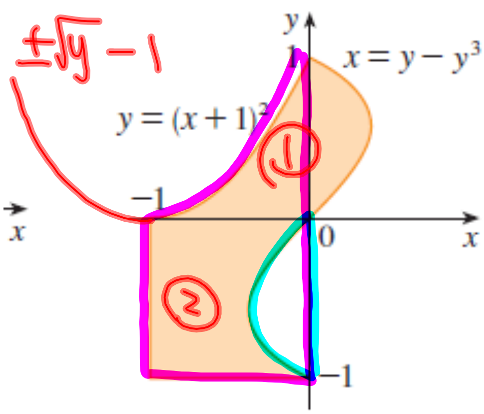
52. $\iint_D y\, dA$



① $\sqrt{y} - 1 \le x \le y - y^3$

$0 \le y \le 1$

$$\int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y\, dx\, dy$$

$$= \int_0^1 y(y - y^3 - \sqrt{y} + 1)\, dy$$

$$= \frac{y^3}{3} - \frac{y^5}{5} - \frac{y^{5/2}}{5/2} + \frac{y^2}{2} \Big]_0^1 = \frac{1}{3} - \frac{1}{5} - \frac{2}{5} + \frac{1}{2}$$

$$= \frac{5}{6} - \frac{3}{5} = \frac{7}{30}$$

$-1 \le x \le y - y^3$

$-1 \le y \le 0$

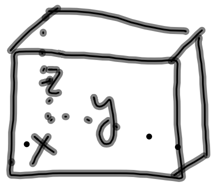
$$\int_{-1}^0 \int_{-1}^{y-y^3} y\, dx\, dy$$

$$= \int_{-1}^0 y(y - y^3 + 1)\, dy = \frac{y^3}{3} - \frac{y^5}{5} + \frac{y^2}{2} \Big]_{-1}^0 = -\left(-\frac{1}{3} + \frac{1}{5} + \frac{1}{2}\right)$$

$$= -\frac{11}{30}$$

$$\boxed{-\frac{4}{30}}$$

40. Find the maximum and minimum volumes of a rectangular box whose surface area is 1500 cm^2 and whose total edge length is 200 cm.



$4x+4y+4z=200$

$x+y+z=50$

min. surf.

$x=50-y-z$

SA= $2xy+2xz+2yz=\cancel{1500}$

$xy+xz+yz=750$

$\Delta f=\langle 1,1,1\rangle$

$\Delta h=\langle y+z, x+z, x+y\rangle$

$y+z=\lambda\cdot 1$

$x+z=\lambda\cdot 1$

$x+y=\lambda\cdot 1$

$\cancel{x}+z=\cancel{x}+y$ & $x+y=y+z$

$z=y$ $\quad$ $x=z$

$\therefore x=y=z$

$x+x+x=50$

$x=y=z=\frac{50}{3}$

max $V=\left(\frac{50}{3}\right)^3$